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Research article

MATHEMATICAL MODEL TO PREDICT ADSORPTION RATE OF POTASSIUM INFLUENCED BY PERMEABILITY IN LATERITIC AND SILTY SOIL FORMATION IN COASTAL AREA OF EAGLE ISLAND, PORT HARCOURT, NIGER DELTA OF NIGERIA

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Abstract

The adsorption rate in the lateritic and silty formation has been expressed, the concept were develop to monitor the rate of adsorption in the coastal location, several application to monitor these substance in the coastal formation were applied but proof abortive, the concept were confirmed to have failed due to lack of thorough information that would have ensure quality results. To solve these challenging problem, mathematical model were find appropriate for the study, the model were develop through the governing equation generated to monitor the adsorption rate in the coastal location. The developed mathematical equations were derived considering all the influential variables in the system. The expression from the governing equation will be applied to ensure the rate of adsorption are determine in lateritic and silty formation, the rate of permeability were expressed in the system, low degree of permeability coefficient generate high accumulation of the substance in lateritic stratum, while in silty formations the coefficient developed higher coefficient of permeability, this reduce its rates of accumulation, thus increase the rate of microbial population in silty formation, the expressed model will definitely monitor the rate of adsorption in the study area.

Key words: Mathematical model, adsorption rate of potassium, permeability and silty soil formation

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1. Introduction

Over the last few decades deterioration of both the quality and quantity of groundwater has become a global phenomenon, which will further intensify the demand for drinking Water increases (World Bank report, 2006). Numerous severe cases of groundwater contamination with reference to storm water infiltration have been documented worldwide (Lopes and Bender, 1998; Fischer, 2003, Zubair, 2007). Few studies have also been documented nationally on groundwater with reference to major ions, trace elements and bacteriology (Rahman et. al., 1997; Zubair, 1998, Zubair, 2007). However literature is silent on the impact of storm water infiltration into groundwater. In recent years attention on the increasing ionic concentration of traces metals in groundwater as result of storm water infiltration has been studied by various workers (Ku et. al., 1992; Appleyard, 1993; Wild, 1994; Hathhorn and Yonge, 1995; Pitt, 1996; Lopes and Bander, 1998; Fisher, 2003). These have been attributed to human interference, proliferation of industries and recent agriculture practices in urban areas where storm water flow recharges the aquifer system and thus degrading the water quality. It is often difficult to determine the exact source of major ions pollutants (Ford, 1990), because there are many potential sources of groundwater contamination including urban storm water runoff. Storm water infiltration in urban areas is cause of concern with regard to the risk of groundwater pollution (Mikkelsen et. al., 1994, Zubair, 2007). Storm water infiltration has been shown to affect groundwater quality and quantity (Pitt, 1996; Fisher, 2003). Contaminants present in urban storm water include volatile organic compound, pesticides, nutrients, and trace elements (Fisher, 2003). This can originate at the land surface or in the atmosphere (Lopes and Bender, 1998). Some constituents either volatize during storage or sorbs to the particulate matter (Mikkelsen et. al., 1996) and are not transported to the water table; however, are more persistent, and may threaten groundwater quality. Malmquist & Hard (1981) studied the impacts from sub surface infiltration at three sites in Sweden and concluded that storm water infiltration affects the groundwater quality to a small extent. In a vast majority of developing countries, fast growing populations combined with poor living conditions in rural areas have forced many people to migrate to cities in search of better living conditions. This has led to a dramatic expansion of most of the major cities throughout developing countries, mainly via the uncontrolled growth of slums or squatter settlements on their fringes (Khadam, 1988; Pathak, 1994; Wang et. al., 2003). Nitrogen is one of the most abundant elements in the Earth’s biosphere and one of the six elements (C, H, O, N, P, and S) that are the major constituents of living tissue. Nitrogen gas (N2) comprises approximately 78% of the Earth’s atmosphere, but this is largely unavailable as a nitrogen source for most living organisms. Consequently, nitrogen availability in all ecosystems is largely dependent on inputs of biologically available nitrogen from external sources or internal cycling of nitrogenous compounds into biologically available forms. Nitrogen often limits biological production in estuaries, oceans, and many terrestrial systems (Schlesinger 1997), and can be limiting in lakes (White et al. 1985, Dodds et al. 1993), streams (Grimm and Fisher 1986, Hill and Knight 1988), and wetlands (Shaver et al. 1986). However, excess nitrogen can have detrimental effects. For example, excess nitrogen or nitrogen saturation (nitrogen losses approaching nitrogen inputs) can lead to increased losses of nutrient cations and increased soil and water acidity in forest ecosystems (Vitousek et al. 1997). In aquatic habitats, excess nitrogen can lead to eutrophication or levels of ammonia (NH3), nitrite (NO2 -), and nitrate (NO3 -) toxic to humans, livestock, and wildlife (Cairns et al. 1990, Carpenter et al. 1998, Marco et al. 1999, Soupir, et al 2006). Land application of waste from confined animal production facilities is an effective method of disposing of animal waste while supplying nutrients to crops and pastureland. However, it has been well-documented that runoff from agricultural livestock and poultry litter applied areas is a source of fecal contamination in water (Crowther *et al.*, 2002; Edwards *et al.*, 1994, 2000; Gerba and Smith, 2005; Tian *et al.*, 2002). The EPA's National Water Quality Inventory report (USEPA, 2000) identified bacteria as the leading cause of impairments in rivers and streams in the United States and agricultural practices were identified as the leading source of all bacterial impairments.

2. Theoretical Background

The adsorption rate of potassium was confirmed in the study location through risk analysis carried out. High depositions of potassium were confirmed through the risk analysis and the adsorption rates were also confirmed through the same analysis also. Adsorption rate in the soil formation were based on permeability influence on the soil formation. Various coefficient of permeability were confirmed to have influenced the rate of adsorption of potassium in the study location. Potassium is one of the substrate utilization of microbial growth which increases microbial population in soil and water environment. This condition implies that depositions of potassium are the major source of microbial increase in lateritic and silty formations. The study location deposited these two types of formations; the lateritic soil has low permeability while the silty formation has a higher permeability with hydraulic conductivity and unconfined bed. The adsorption rate of potassium accumulated more between organic and lateritic soil. High rate of these depositions can also be attributed to environmental influence. These are through climatic a condition that reflects high rain intensities subjecting the soil to high degree of saturation. The accumulation of microelements' in the study location experiences more penetration through of the two lateritic and silty formation, low rate of permeability and constant regeneration of the substance develop adsorption of potassium in the coastal formation , the study is imperative because the rate of adsorption were found to be on exponential rate between these two soil formation. The rates of adsorption develop high degree of microbial growth, thus pollutes ground water aquifers in the study locations.

Governing equation

$$\theta m \frac{\partial Cs}{\partial t} = K \frac{\partial Cs}{\partial X}\left[C^{\omega}{}_{p} - C^{\omega}{}_{p}\right] \quad \ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

Equation is the governing equation that express the rate of accumulation of potassium in silty and lateritic soil, the two formation were monitored on deposition of potassium in the coastal environment of the Niger delta, studies from microelement deposition were found to disperse to a very large extent in the coastal environment, the developed concept could not generate a better solution to the current challenges of accumulation of this substance in lateritic and salty formation, in the coastal environment. The developed governing equations were expressed through the influential parameter in the system, the governing equations are stated below.

Applying Laplace transformation into equation (2)

$$\frac{\partial Cs}{\partial t} = S^1C_{(t)} - C_{(o)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial Cs}{\partial t} = S^1C_{(x)} - C_{(o)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$Cs = C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\theta m\left[S^1C_{(t)} - C_{(o)}\right] - K\left[SC_{(t)} - C_{(x)}\right] - C^{\omega}{}_p - C^{\omega}{}_p \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\theta m\left[SC_{(t)} - \theta m S_{(t)} - C_{(o)} - K\,SC_{(x)} + K\,C_{(o)} - C^{\omega}{}_p - C^{\omega}{}_p\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Considering the following boundary condition at

$$t = 0,\ C^1{}_{(o)} = C^1{}_{(o)} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$C_{(t)} - \theta m S^1 - \theta m S - KS = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$C_{(t)} \neq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

But considering the boundary condition at

$$t = 0,\ C^1{}_{(o)} = C_{(o)} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

The expression that will ensure that the rates of adsorption are determined in the coastal environments the application of Laplace transformation. The concept are applied to transform the parameters in the system to the mathematical tools that will develop the expressed product in the system and crate a platform for these variables to express the subjective relation to each other in the system. Applying the concept, establishments of boundary values were suitable in the system, this is to state there limits base on the behaviour of the substance on the accumulating process in the lateritic and silty formation, therefore boundary value were established and the expressed equation will ensure that the concepts are applied base on the boundary condition stated in the system.

$$\theta m\,S^1C_{(t)} - K\,SC_{(x)} - C^{\omega}{}_p - C^{\omega}{}_p\,C_{(x)} = \theta m\,SC_{(o)} + \theta m\,C_o + KC_o \qquad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\left[\theta m\,S^1 - KS - C^s{}_{\omega} - C^s{}_{\omega}\right]C_{(t)} = \left[\theta m\,S + \theta m + K\right]C_o \qquad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$C_{(t)} = \frac{\theta m\,S + \theta m + K}{\theta m\,S + \theta m\,S - C^s{}_{\omega} - C^s{}_{\omega}}C_o \qquad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

The application of Laplace transformation continue by expressing the parameter to their various function and establishing relationship with one another, this method are applied through the expression stated between equation [11] to [13]. Subject to this relation, the rate of adsorptions in the system are determined by the rate of regeneration of the substance between the lateritic and silty formation, the formation experience exponential phase in hydraulic conductivity, where silty formation accumulate the substance lower than the lateritic soil formation due to high rate of hydraulic conductivity and void ratio in the coastal formation

Applying quadratic expression we have

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \ldots\ldots\ldots\ldots\ldots.. (14)$$

The rate of accumulation of potassium in lateritic and silty formations are on the exponential phase in the study location, that why it is imperatives that such study should be carried out in other to monitor the rate of accumulation, base on the this condition quadratic expression were find suitable in the study, this concept were applied to monitor the accumulative Substance in exponential phase.

Where a = θm, b = $-K$, c = $C^{\omega}{}_{p} - C^{\omega}{}_{p}$

$$\frac{-K \pm \sqrt{K^2 - 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m} \qquad \ldots\ldots\ldots\ldots\ldots.. (15)$$

$$S_1 = \frac{K \pm \sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m} \qquad \ldots\ldots\ldots\ldots\ldots.. (16)$$

$$S_2 = \frac{K - \sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m} \qquad \ldots\ldots\ldots\ldots\ldots.. (17)$$

$$S_1 = \frac{K + \sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m} \quad + \quad S_2 = \frac{K - \sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m}$$

$$e^{\left[u + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m}\right] \frac{L}{V} + \left[-K - K\, \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m}\right]} \qquad \ldots\ldots\ldots\ldots\ldots.. (18)$$

Apply Laplace inverse of the equation were considered because the process of adsorptions the lateritic formation develop lower hydraulic conductivity than silty formation because there hydraulic conductivity are more, thus inverse Laplace were applied to express this condition in the system as stated from below.

$$C_{(t)} = \left[\theta m/_{t} + \theta m + K \right] C_o \quad e^{\left[K + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m}\right] t}$$

$$e^{\left[K - \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_{p} - C^{\omega}{}_{p}}}{2\theta m}\right] t} \qquad \ldots\ldots\ldots\ldots\ldots.. (19)$$

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$$\left[C\,[L,V]\ {}^{\theta m}\!/_{L/_V} + \theta m + K\right] C_o\ e^{\left[K + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V}}$$

Considering the following boundary conditions at

$$t = 0,\ C^1{}_{(o)} = C_{(o)} = C_o \qquad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

The expression from [19] to 20] defined the period rate of accumulation in the formation, the substance are deposited in two ways natural origin and manmade activity, but the focus of the studies centered more on manmade activates, this condition are expressed in the system mathematically on time, [T] the expressed equation between [19] and [20] define the function of the parameter [T] i.e. time of adsorption in lateritic and silty formations, these conditions were considered in the system due to the rate of variations establish in the coastal locations. The concept was adopted to ensure that the system express period of accumulation between the two formations.

$$C_{(t)} = \left[{}^{\theta m}\!/_t + \theta m + K\right] C_o\ e^{\left[K + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V} - \left[K - \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V}} \qquad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

$$\text{At } C^1{}_{(o)} = t \neq 0 \qquad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

Again

$C_{(o)} = C_{(o)}$ So that $C_o = [\theta m + K] C_o\,[1 + 1]$ i.e.

$$0 = [\theta m + K] \qquad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$\Rightarrow \theta m + K = 0 \qquad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

So that we have

$$C_{(t)} = \left[2\frac{\theta m}{t}\right] C_o\ e^{\left[K + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V} +}$$

$$\left[K - \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V} \qquad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

Suncidal application

However, $e^x - e^x = 2Cos\ x,$ therefore, we have

$$C_{(t)} = \left[2\frac{\theta m}{t}\right] C_o\, Cos \left[K + \frac{\sqrt{K^2 + 4\theta m\, C^{\omega}{}_p - C^{\omega}{}_p}}{2\theta m}\right]\frac{L}{V} \qquad \ldots\ldots \quad (26)$$

The expression in [26] is the final model to predict adsorption rate of potassium influenced by permeability in lateritic and silty formation. The model were derived through the governing equation, the model was derived considering various condition that cause the adsorption of potassium in lateritic and silty formation, mathematical model were develop, this was applied to monitor the rate of adsorption including period of absorption in the coastal location. The influence of permeability were also monitored in the system, the rate of permeability in the system were expressed because lateritic and silty formations deposit various coefficients in the study location, such condition were expressed in the system, these conditions determine the rate of accumulation of the substance at various formation. The expressed mathematical model has thoroughly reacted with numerous functions that develop the adsorption rate of potassium under the influence of permeability in lateritic and silty formation in the study area.

4. Conclusion

The rate of potassium adsorption in lateritic and silty formations has been expressed; the study was carried out to monitor the rate of adsorption in the coastal formation, the study considered several challenges that develop different stratification in the formations. Numerous influence were expressed in the system, the condition has been found through risk assessment carried out in the study location, base on this condition mathematical model were found suitable to establish a model that will determine the rate of adsorption in lateritic and silty formation, the adsorption rate were found to be influenced by high degree of permeability, the deposition of permeability were very high, this base on the stratification of the coastal formation, this condition influenced the adsorption rate of potassium and increase the rate of concentration of microbes in coastal formation. The expressions were developed base on the influence from high degree of adsorption,they are considered in other to express their various function in the developed model, the final express model will monitor the rate of adsorption in lateritic and silty formation in the study area.

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